

## Design of Combined Footings

Summary of combined footing design is shown in the following steps.

**1- Select a trial footing depth.**

**2- Establish the required base area of the footing:**

Uniform soil pressure is achieved by making the resultant of the service column loads coincide with the centroid of the footing base.

**3- Evaluate the net factored soil pressure.**

**4- Check footing thickness for punching shear.**

**5- Draw S.F.D and B.M.D for footing:**

The shear force and bending moment diagrams for the footing are to be drawn, considering the footing as an inverted beam subjected to distributed load. Column loads may be considered as concentrated loads to simplify the analysis.

**6- Check footing thickness for beam shear.**

Beam shear capacity is checked against critical factored shear evaluated from S.F.D. Depth of footing may be increased to satisfy beam shear requirement.

**7- Compute the area of flexural reinforcement:**

Flexural reinforcement required in the longitudinal direction is evaluated.

Furthermore, the areas of flexural reinforcement under columns in the transverse direction are computed. In computing this reinforcement, it will be assumed that each column load is uniformly distributed over a band centered on the column and having a width extending a distance  $d/2$  from its faces. This area of reinforcement is to be uniformly distributed across the band width  $(C_2+d)$ , and the rest of the footing is to be provided with shrinkage and temperature reinforcement.

**8- Check bearing strength of column and footing concrete.**

**9- Check for anchorage of the reinforcement.**

**10- Prepare neat design drawings showing footing dimensions and provided reinforcement.**

**Example (11.4):**

Design an appropriate footing/footings to support two columns *A* and *B* spaced at distance 2.1 *m* center-to-center, as shown in Figure 11.13.a is 20 *cm* × 30 *cm* and carries a dead load of 20 *tons* and a live load of 10 *tons*. Column *B* is 20 *cm* × 40 *cm* in cross section but carries a dead load of 30 *tons* and a live load of 15 *tons*. Width of footing is not to exceed 1.0 *m*, and there is no property line restriction.

Use  $f'_c = 300 \text{ kg/cm}^2$ ,  $f_y = 4200 \text{ kg/cm}^2$ ,  $q_{all}(\text{gross}) = 2.0 \text{ kg/cm}^2$ ,  $g_{soil} = 1.7 \text{ t/m}^3$ , and  $D_f = 2.0 \text{ m}$ .

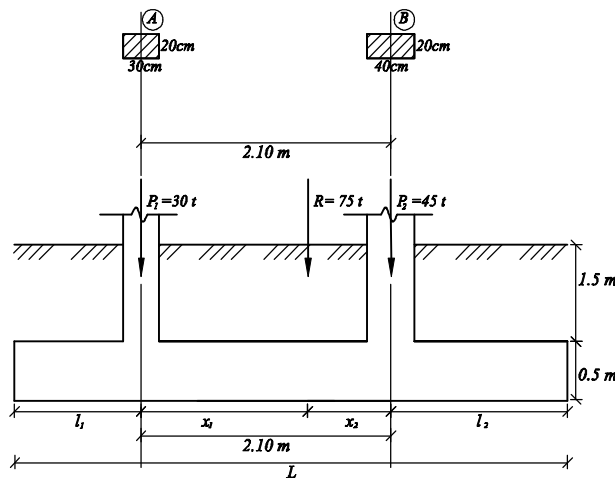


Figure 11.13.a: Footing dimensions

**Solution:**

**1- Select a trial footing depth:**

Assume that the footing is 50 cm thick.

**2- Establish the required base area of the footing:**

$$q_{all}(\text{net}) = 20 - 1.5(1.7) - 0.5(2.5) = 16.2 \text{ t/m}^2$$

If isolated footings are to be used,

$$A_{1req} = \frac{P_A}{q_{all}(\text{net})} = \frac{30}{16.2} = 1.85 \text{ m}^2$$

$B = 1 \text{ m}$ , and  $L = 1.85 \text{ m}$ .

$$A_{2req} = \frac{P_B}{q_{all}(\text{net})} = \frac{45}{16.2} = 2.78 \text{ m}^2$$

$B = 1 \text{ m}$ , and  $L = 2.80 \text{ m}$ .

Overlapping of the two footings occurs, ruling out isolated footings as appropriate of footing types. A combined footing will be used instead.

$$A_{req} = \frac{P_A + P_B}{q_{all}(\text{net})}$$

$$A_{req} = \frac{20 + 10 + 30 + 15}{16.2} = 4.63 \text{ m}^2$$

To locate the resultant of the column forces, replace the column load system with a resultant force system, or

$$P_B (2.1) = R(x_1), \text{ or } 45(2.1) = (30 + 45)(x_1) \text{ and } x_1 = 1.26 \text{ m}$$

$$x_2 = 2.1 - 1.26 = 0.84 \text{ m}$$

$$l_1 + x_1 = l_2 + x_2, \text{ or}$$

$$l_2 - l_1 = x_1 - x_2 = 0.42 \text{ m} \quad (\text{a})$$

$$\text{Let } B = 1 \text{ m, length of footing } L = \frac{4.63}{1.0} = 4.63 \text{ m, taken as } 4.65 \text{ m}$$

$$l_1 + l_2 + 2.1 = 4.65 \text{ m and}$$

$$l_1 + l_2 = 2.55 \text{ m} \quad (\text{b})$$

Adding (a) and (b) gives,  $l_2 = 1.485 \text{ m}$  and  $l_1 = 1.065 \text{ m}$

### 3- Evaluate the net factored soil pressure:

$$P_{Au} = 1.20(20) + 1.60(10) = 40 \text{ tons}$$

$$P_{Bu} = 1.20(30) + 1.60(15) = 60 \text{ tons}$$

$$q_u (\text{net}) = \frac{P_{Au} + P_{Bu}}{LB} = \frac{40 + 60}{4.65(1.0)} = 21.51 \text{ t/m}^2$$

### 4- Check footing thickness for punching shear:

#### Column B:

Effective depth  $d = 50 - 7.5 - 0.80 = 41.7 \text{ cm}$  (lower layer)

The factored shear force

$$V_u = 60 - (21.51)(0.817)(0.617) = 49.16 \text{ tons}$$

$$b_o = 2(81.7 + 61.7) = 286.8 \text{ cm}$$

$\Phi V_c$  is the smallest of:

$$\begin{aligned} F V_c &= 0.53 F \sqrt{f'_c} \left( 1 + \frac{2}{b} \right) 1 b_o d \\ &= 0.53 (0.75) \sqrt{300} \left( 1 + \frac{2}{40/20} \right) (286.8)(41.7) / 1000 = 164.68 \text{ tons} \end{aligned}$$

$$\begin{aligned} F V_c &= 1 F \sqrt{f'_c} b_o d \\ &= 0.75 \sqrt{300} (286.8)(41.7) / 1000 = 155.36 \text{ tons} \end{aligned}$$

$$\begin{aligned} F V_c &= 0.27 F \left( \frac{a_s d}{b_o} + 2 \right) 1 \sqrt{f'_c} b_o d \\ &= 0.27 (0.75) \left( \frac{40(41.7)}{286.8} + 2 \right) \sqrt{300} (286.8)(41.7) / 1000 = 327.85 \text{ tons} \end{aligned}$$

$$F V_c = 155.36 \text{ tons} > 49.16 \text{ tons}$$

**Column A:**

The factored shear force

$$V_u = 40 - (21.51)(0.717)(0.617) = 30.48 \text{ tons}$$

$$b_o = 2(71.7 + 61.7) = 266.8 \text{ cm}$$

$\Phi V_c$  is the smallest of:

$$\begin{aligned} F V_c &= 0.53 F \sqrt{f'_c} \left( 1 + \frac{2}{b} \right) l b_o d \\ &= 0.53 (0.75) \sqrt{300} \left( 1 + \frac{2}{30/20} \right) (266.8)(41.7) / 1000 = 178.73 \text{ tons} \end{aligned}$$

$$\begin{aligned} F V_c &= 1 F \sqrt{f'_c} b_o d \\ &= 0.75 \sqrt{300} (266.8) (41.7) / 1000 = 144.53 \text{ tons} \end{aligned}$$

$$\begin{aligned} F V_c &= 0.27 F \left( \frac{a_s d}{b_o} + 2 \right) l \sqrt{f'_c} b_o d \\ &= 0.27 (0.75) \left( \frac{40(41.7)}{266.8} + 2 \right) \sqrt{300} (266.8) (41.7) / 1000 = 322 \text{ tons} \end{aligned}$$

$$F V_c = 144.53 \text{ tons} > 30.48 \text{ tons}$$

**5- Draw S.F.D and B.M.D for footing:**

S.F.D and B.M.D are shown in Figures 11.13.d and 11.13.e.

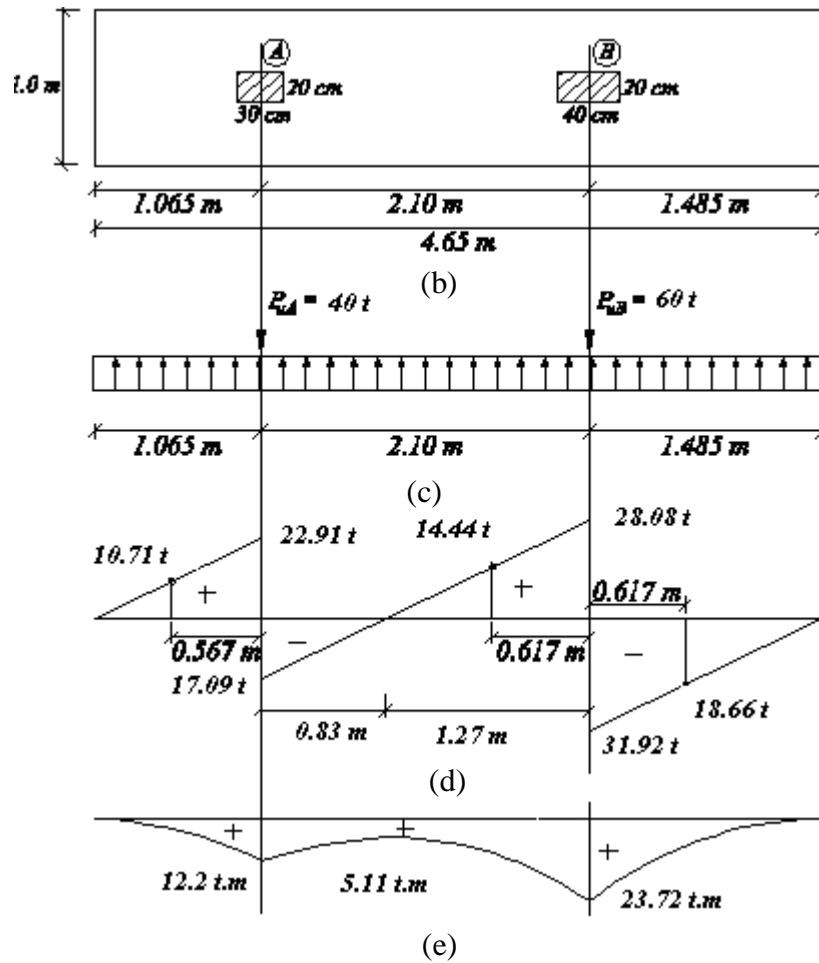


Figure 11.13: (continued); (b) Footing dimensions; (c) factored soil pressure; (d) shearing force diagram; (e) bending moment diagram

**6- Check footing thickness for beam shear:**

$$FV_c = 0.75(0.53)\sqrt{300}(100)(41.7)/1000 = 28.71 \text{ tons}$$

Maximum factored shear force  $V_u$  is located at distance  $d$  from the exterior face of column B,

$$V_u = 18.66 \text{ tons} < 28.71 \text{ tons} \text{ O.K.}$$

**7- Compute the area of flexural reinforcement:**

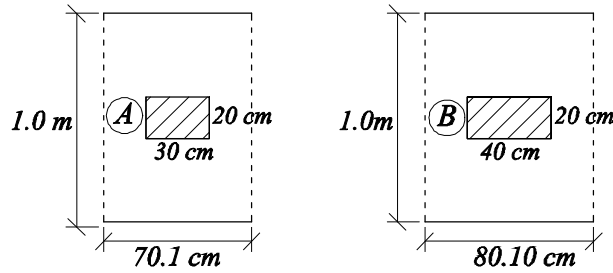
**a. Bottom longitudinal reinforcement:**

$$r = \frac{0.85(300)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(23.72)}{0.9(100)(41.7)^2(300)}} \right] = 0.0037$$

$$A_s = 0.0037(100)(41.7) = 15.43 \text{ cm}^2, \text{ use } 8 \text{ f } 16 \text{ mm}$$

**b. Transverse reinforcement:**

Effective depth  $d = 50 - 7.5 - 1.6 - 0.80 = 40.1 \text{ cm}$  (upper layer)



**Figure 11.13.f: Transverse reinforcement (width of strips)**

**Under Column A:**

$$M_{uA} = \frac{40}{(0.701)(1.0)} \frac{(0.701)}{2} \left( \frac{1-0.2}{2} \right)^2 = 3.2 \text{ t.m}$$

$$r = \frac{0.85(300)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(3.2)}{0.9(70.1)(40.1)^2(300)}} \right] = 0.00076$$

so use  $r = r_{min} = 0.0018$

$$A_s = 0.0018(70.1)(50) = 6.31 \text{ cm}^2, \text{ use } 5 \text{ f } 14 \text{ mm}$$

**Under Column B:**

$$M_{uB} = \frac{60}{(0.801)(1.0)} \frac{(0.801)}{2} \left( \frac{1-0.2}{2} \right)^2 = 4.80 \text{ t.m}$$

$$r = \frac{0.85(300)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(4.8)}{0.9(80.1)(40.1)^2(300)}} \right] = 0.0010$$

so use  $r = r_{min} = 0.0018$

$$A_s = 0.0018(80.1)(50) = 7.21 \text{ cm}^2, \text{ use } 5 \text{ f } 14 \text{ mm}$$

Shrinkage Reinforcement in the short direction (bottom side):

$$A_s = 0.0018(465 - 80.1 - 70.1)(50) = 28.33 \text{ cm}^2, \text{ use } 26 \text{ f } 12 \text{ mm}$$

**8- Check bearing strength of column and footing concrete:**

$$F P_{An} = 0.65(0.85)(300)(20)(30)/1000 = 199.45 \text{ tons} > 40 \text{ tons}$$

Use minimum dowel reinforcement,  $A_s = 0.005(20)(30) = 3.0 \text{ cm}^2$

$$F P_{Bn} = 0.65(0.85)(300)(20)(40)/1000 = 132.6 \text{ tons} > 60 \text{ tons}$$

Use minimum dowel reinforcement,  $A_s = 0.005(20)(40) = 4.0 \text{ cm}^2$

**9- Check for anchorage of the reinforcement:**

**Bottom longitudinal reinforcement (f 16 mm)::**

$$y_t = y_e = l = 1 \text{ and } y_s = 0.8$$

$c_b$  is the smaller of:

$$7.5 + 0.8 = 8.3 \text{ cm, or } \frac{100 - 15 - 1.6}{8(2)} = 5.21 \text{ cm, i.e., } c_b = 5.21 \text{ cm}$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{5.21 + 0}{1.6} = 3.26 > 2.5, \text{ take it equal to } 2.5$$

$$l_d = \frac{1.6(0.8)(4200)}{3.5(2.5)\sqrt{300}} = 35.47 \text{ cm}$$

$$\text{Available length} = 106.5 - 7.5 = 99 \text{ cm} > 35.47 \text{ cm}$$

Similarly, bottom transverse reinforcement is to be checked for anchorage, and will be found satisfactory.

### 10-Prepare neat design drawings showing footing dimensions and provided reinforcement:

Detailed design drawings are shown in Figure 11.13.g.

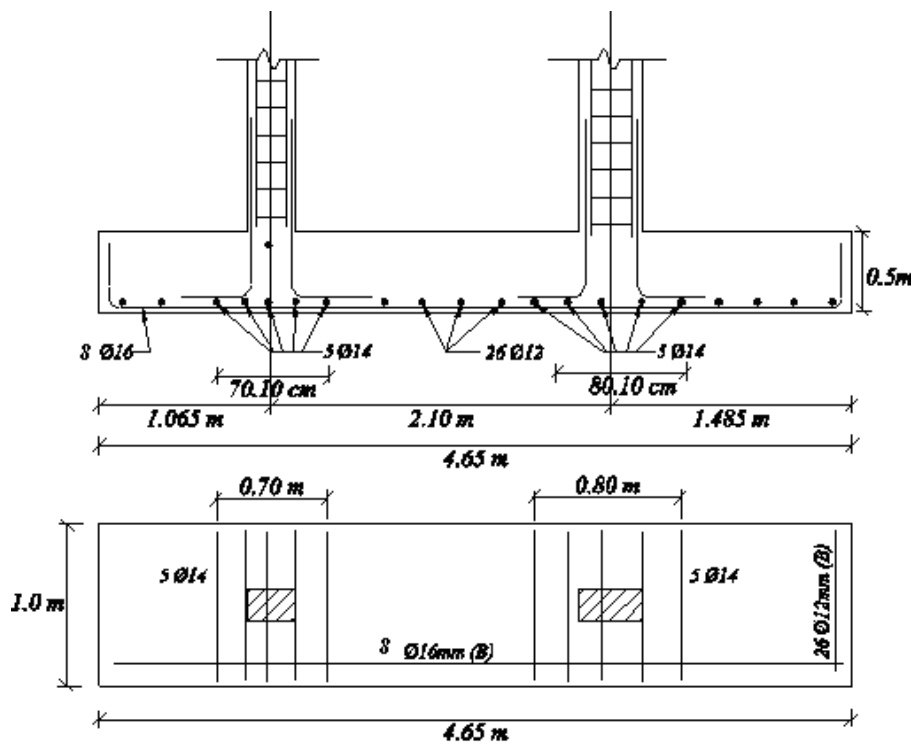
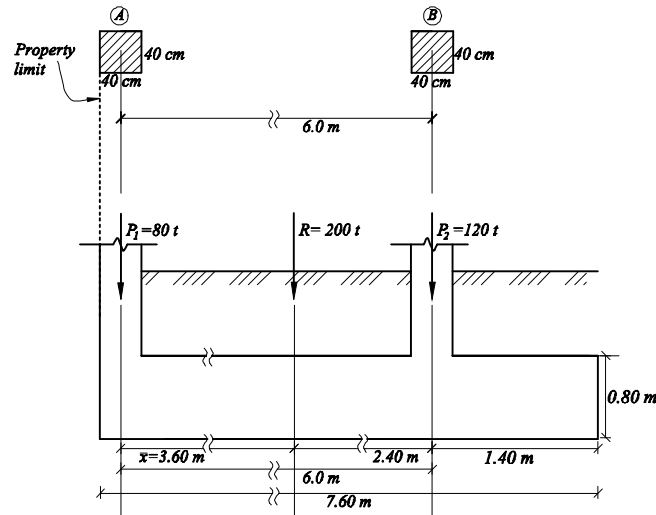


Figure 11.13.g: Design drawings

**Example (11.5):**

Design a combined footing, to support two columns A and B spaced at distance 6.0 m center-to-center as shown in Figure 11.14.a. Column A is 40 cm × 40 cm and carries a dead load of 50 tons and a live load of 30 tons. Column B is also 40 cm × 40 cm in cross section but carries a dead load of 70 tons and a live load of 50 tons.

Use  $f'_c = 250 \text{ kg/cm}^2$ ,  $f_y = 4200 \text{ kg/cm}^2$ , and  $q_{all}(\text{net}) = 1.50 \text{ kg/cm}^2$ .



**Figure 11.14.a: Footing dimensions**

**Solution:****1- Select a trial footing depth:**

Let footing thickness  $h = 80 \text{ cm}$

**2- Establish the required base area of the footing:**

$$A_{req} = \frac{P_A + P_B}{q_{all}(\text{net})}$$

$$A_{req} = \frac{50 + 30 + 70 + 50}{15.0} = 13.33 \text{ m}^2$$

To locate the resultant of the column forces,

$$P_B (6) = R(\bar{x}), \text{ or } 120 (6) = (80 + 120)(\bar{x}) \text{ and } \bar{x} = 3.60 \text{ m}$$

$$\text{Length of footing } L = 2(3.60 + 0.20) = 7.60 \text{ m}$$

$$\text{Width of footing } B = \frac{13.33}{7.60} = 1.75 \text{ m, taken as } 1.80 \text{ m.}$$

**3- Evaluate the net factored soil pressure:**

$$P_{Au} = 1.20(50) + 1.60(30) = 108 \text{ tons}$$



$$P_{Bu} = 1.20(70) + 1.60(50) = 164 \text{ tons}$$

$$q_u (\text{net}) = \frac{P_{Au} + P_{Bu}}{LB} = \frac{108 + 164}{7.60(1.80)} = 19.88 \text{ t/m}^2$$

#### 4- Check footing thickness for punching shear:

Effective depth  $d = 80 - 7.5 - 0.90 = 71.60 \text{ cm}$  (lower layer)

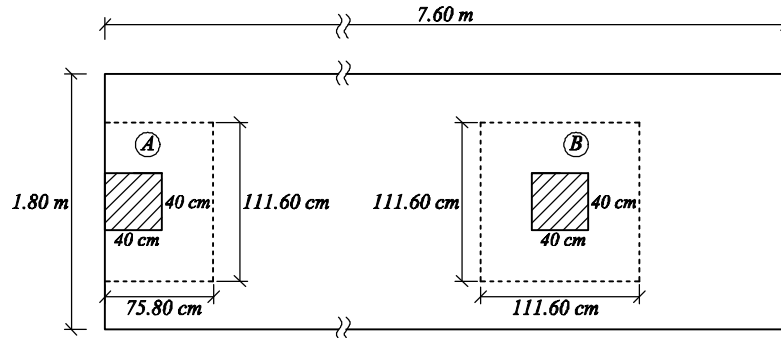


Figure 11.14.b: Critical sections for punching shear

#### Column A:

The factored shear force  $V_u = 108 - (19.88)(1.116)(0.758) = 91.18 \text{ tons}$

$$b_o = 2(40 + 35.8) + 40 + 71.6 = 263.2 \text{ cm}$$

$\Phi V_c$  is the smallest of:

$$F V_c = 0.53 F \sqrt{f'_c} \left(1 + \frac{2}{b}\right) I b_o d$$

$$= 0.53(0.75) \sqrt{250} \left(1 + \frac{2}{40/40}\right) (263.2)(71.6) / 1000 = 355.33 \text{ tons}$$

$$F V_c = 1 F \sqrt{f'_c} b_o d$$

$$= 0.75 \sqrt{250} (263.2) (71.6) / 1000 = 223.48 \text{ tons}$$

$$F V_c = 0.27 F \left(\frac{a_s d}{b_o} + 2\right) I \sqrt{f'_c} b_o d$$

$$= 0.27(0.75) \left(\frac{30(71.6)}{263.2} + 2\right) \sqrt{250} (263.2) (71.6) / 1000 = 613.10 \text{ tons}$$

$$F V_c = 223.48 \text{ tons} > 91.18 \text{ tons}$$

#### Column B:

The factored shear force  $V_u = 164 - (19.88)(1.116)^2 = 139.24 \text{ tons}$

$$b_o = 4(40 + 71.6) = 446.4 \text{ cm}$$

$\Phi V_c$  is the smallest of:

$$F V_c = 0.53 F \sqrt{f'_c} \left( 1 + \frac{2}{b} \right) l b_o d$$

$$= 0.53 (0.75) \sqrt{250} \left( 1 + \frac{2}{40/40} \right) (446.4)(71.6) / 1000 = 602.65 \text{ tons}$$

$$F V_c = 1 F \sqrt{f'_c} b_o d$$

$$= 0.75 \sqrt{250} (446.4)(71.6) / 1000 = 379.02 \text{ tons}$$

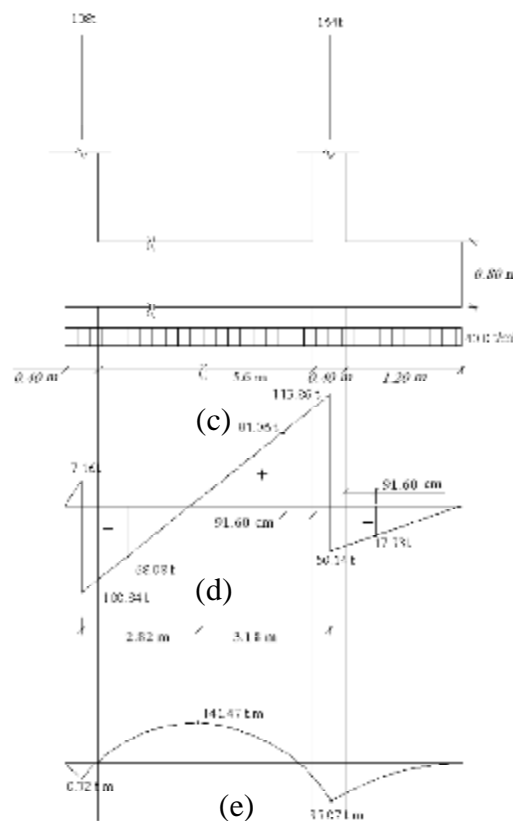
$$F V_c = 0.27 F \left( \frac{a_s d}{b_o} + 2 \right) l \sqrt{f'_c} b_o d$$

$$= 0.27 (0.75) \left( \frac{40(71.6)}{446.4} + 2 \right) \sqrt{250} (446.4)(71.6) / 1000 = 861.24 \text{ tons}$$

$$F V_c = 379.02 \text{ tons} > 139.24 \text{ tons}$$

### 5- Draw S.F.D and B.M.D for footing:

S.F.D and B.M.D are shown in Figures 11.14.e and 11.14.f.



**Figure 11.14: (continued); (c) Factored soil pressure; (d) shearing force diagram (not to scale); (e) bending moment diagram (not to scale)**

### 6- Check footing thickness for beam shear:

Effective depth  $d = 80 - 7.5 - 0.90 = 71.60 \text{ cm}$  (lower layer)

$$F V_c = 0.75 (0.53) \sqrt{250} (180)(71.60) / 1000 = 81.0 \text{ tons}$$

Maximum factored shear force  $V_u$  is located at distance  $d$  from the left face of column B,

$$V_u = 81.06 \text{ tons} \cong 81.0 \text{ tons}$$

**7- Compute the areas of flexural reinforcement:**

**a- Top longitudinal reinforcement:**

$$r = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5 (141.47)}{(0.9)180(71.6)^2 (250)}} \right] = 0.00423$$

$$A_s = 0.00423(180)(71.6) = 54.52 \text{ cm}^2, \text{ use } 18 \text{ f } 20 \text{ mm}$$

**b- Bottom longitudinal reinforcement:**

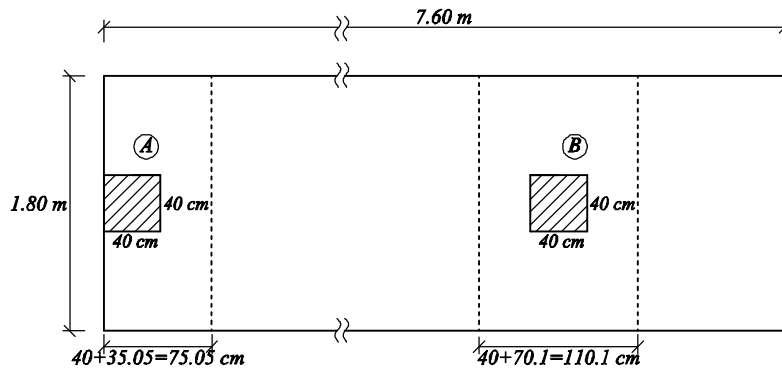
$$r = 0.0010 < r_{\min}, \text{ so use } r = r_{\min} = 0.0018$$

$$A_s = 0.0018(180)(80) = 25.92 \text{ cm}^2, \text{ use } 13 \text{ f } 16 \text{ mm}$$

**c- Short direction:**

**Under Column A:**

Effective depth  $d = 80 - 7.5 - 1.6 - 0.8 = 70.10 \text{ cm}$  (upper layer)



**Figure 11.14.f: Transverse reinforcement (width of strip)**

$$M_{Au} = \frac{108}{(0.7505)(1.80)} \frac{(0.7505) \left( \frac{1.8-0.4}{2} \right)^2}{2} = 14.70 \text{ t.m}$$

$$r = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5 (14.70)}{(75.05)(0.9)(70.1)^2 (250)}} \right] = 0.0011$$

Use  $r = r_{\min} = 0.0018$

$$A_s = 0.0018(75.05)(80) = 10.81 \text{ cm}^2, \text{ use } 6 \text{ f } 16 \text{ mm}$$

**Under Column B:**

$$M_{Bu} = \frac{164}{(1.101)(1.80)} \frac{(1.101) \left( \frac{1.8-0.4}{2} \right)^2}{2} = 22.32 \text{ t.m}$$

$$r = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5 (22.32)}{0.9(110.1)(70.1)^2 (250)}} \right] = 0.0011$$

$r = 0.0012 < r_{\min}$ , so use  $r = r_{\min} = 0.0018$

$$A_s = 0.0018(110.1)(80) = 15.85 \text{ cm}^2, \text{ use } 8 \text{ f } 16 \text{ mm}$$

Shrinkage Reinforcement in the short direction (bottom side):

$$A_s = 0.0018(760 - 110.1 - 75.05)(80) = 82.78 \text{ cm}^2, \text{ use } 42 \text{ f } 16 \text{ mm}$$

Shrinkage Reinforcement in the short direction (top side):

$$A_s = 0.0018(760)(80) = 109.44 \text{ cm}^2, \text{ use } 55 \text{ f } 16 \text{ mm}$$

### 8- Check bearing strength of column and footing concrete:

$$F P_n = 0.65(0.85)(250)(40)(40) / 1000 = 221 \text{ tons} > 164 \text{ tons}$$

i.e. use minimum dowel reinforcement,  $A_s = 0.005(40)(40) = 8.0 \text{ cm}^2$

### 9- Check for anchorage of the reinforcement:

**a- Top reinforcement ( f20mm ):**

$y_t = 1.3$  since more than 30 cm of fresh concrete is cast below the reinforcement,  $y_e = l = 1$  and

$$y_s = 0.8$$

$c_b$  is the smaller of:

$$7.5 + 1 = 8.5 \text{ cm}, \text{ or } \frac{180 - 7.5 - 7.5 - 2}{19(2)} = 4.29 \text{ cm}, \text{ i.e., } c_b = 4.29 \text{ cm}$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{4.29 + 0}{2.0} = 2.145$$

$$l_d = \frac{2.0(0.8)(1.3)(4200)}{2.145(3.5)\sqrt{250}} = 73.59 \text{ cm}$$

Available development length =  $262.5 + 40 - 7.5 = 295.0 \text{ cm} > 73.59 \text{ cm}$

**b- Bottom reinforcement( f16mm ):**

$y_t = y_e = l = 1$  and  $y_s = 0.8$

$c_b$  is the smaller of:

$$7.5 + 0.8 = 8.3 \text{ cm}, \text{ or } \frac{180 - 7.5 - 7.5 - 1.6}{12(2)} = 6.81 \text{ cm}, \text{ i.e., } c_b = 6.81 \text{ cm}$$

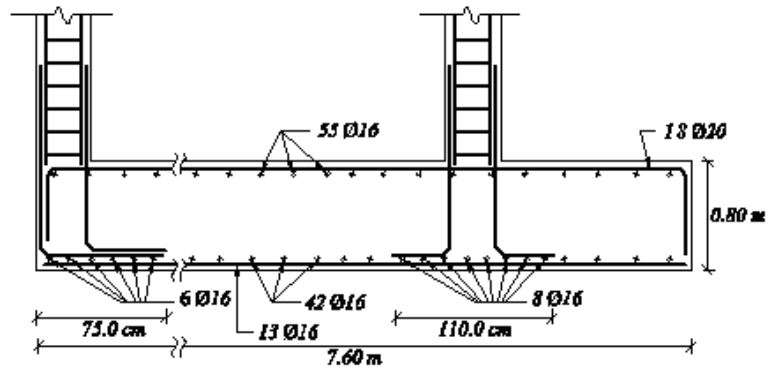
$$\frac{c_b + K_{tr}}{d_b} = \frac{6.81 + 0}{1.6} = 4.26 > 2.5, \text{ take it equal to } 2.5$$

$$l_d = \frac{1.6(0.8)(4200)}{2.5(3.5)\sqrt{250}} = 38.86 \text{ cm}$$

Available length =  $120 + 11.5 - 7.5 = 124 \text{ cm} > 38.86 \text{ cm}$

**10- Prepare neat design drawings showing footing dimensions and provided reinforcement:**

Detailed design drawing is shown in Figure 11.14.g.



**Figure 11.14.g: Design drawing**